The challenge of implementing the Common Core Mathematics Standards

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H. Wu

Overview: The CCSSM vs. TSM

By 2014, the Common Core State Standards in Mathematics (**CCSSM**) will be phased in.

Are they just like all other mathematics standards that have come and gone?

This question must be answered, and it is *No*.

But there is a perception out there—among policy makers, teachers, and textbook publishers—that the CCSSM are *just* another set of standards.

To quote one administrator: "Given the work of [the national professional organizations and those in our state], I doubt there will be significant changes."

Wrong! There ***are*** *significant changes.* Among those, I want to single out the major one.

The major change is not about pedagogical practices or learning strategies (although both will be impacted).

The major change lies in the mathematics. *The CCSSM call for better mathematical content in the classrooms across the land.*

In most standards, the main concern is whether a certain topic is taught in a certain grade or not at all. For example, will students memorize the multiplication table in grade 3? Is grade 8 devoted to Algebra?

From this perspective, a set of *rigorous standards* is one in which the topics thought to be important are taught as early as possible.

Some other standards try to distinguish themselves by *claiming* to emphasize conceptual understanding and reasoning.

Regardless, all these standards have a **common starting point**, namely, *the mathematics that has been embedded in school textbooks for decades.*

For example, the teaching of the addition of fractions takes many forms in different curricula, reform or traditional.

Some emphasize the drawing of pictures to get the answer and de-emphasize algorithms, others do the opposite, emphasize algorithms over picture-drawing.

But they have one flaw in common: neither tells students *what a fraction is* or *what it means to add fractions.*

Without either, no reasoning is possible and no real learning can take place.

Fraction phobia tells you all you need to know.

If the mathematics is defective, there is no glory in teaching it two years ahead of every state and every nation on earth.

Another example: Some books teach the division of fractions by brute force as *invert-and-multiply*, while others simply avoid the general concept of *division* by only doing simplistic divisions with picture-drawing, e.g., $\dfrac{\frac{1}{2}}{\frac{1}{4}}$.

"Ours is not to reason why,

just invert and multiply."

Neither approach gives students the needed understanding that the division of fractions is, conceptually,

- no different from the division of whole numbers,
- no different from the division of real or complex numbers.

We call the mathematics embedded in school textbooks **TSM** (Textbook School Mathematics). We will give many more examples of **TSM** below.

Improvement in math education depends on eliminating **TSM** from the school mathematics curriculum.

The CCSSM are the first set of standards to challenge **TSM** head-on.

This is what sets the CCSSM apart from other standards. This is the significant change that the CCSSM bring to the table.

Getting the math right will not grab the headlines in the New York Times, but we **must** get it right in order to give our students a chance to learn. As the computer dictum goes:

Garbage in, garbage out.

Successful implementation of the CCSSM therefore depends on teaching our students mathematics and not **TSM**.

We need two things to make this happen:

Getting teachers who know **mathematics** *rather than just* **TSM**.

Getting textbooks that are not filled with **TSM**.

Where to find these teachers when the education establishment does not do its job by providing teachers with the requisite *content knowledge?*

Through no fault of their own, teachers' content knowledge has been limited to **TSM**. But they are now called upon to implement the CCSSM whose goal is to banish TSM from schools.

This is the classical *wishful-thinking syndrome* in education: Shout to the whole world what you want, but make no effort to get it done.

What needs to be done: Commit to content-based, *non*-**TSM** professional development (PD) for teachers.

Better textbooks? Not even close.

"Common Core aligned" textbooks from major publishers have been around even before the CCSSM were released, but those I have seen continue to vigorously promote **TSM** as never before.

Implementation of the CCSSM already has two strikes against it before it gets started. What lies ahead can only be “blood, toil, tears, and sweat.”

You have not heard these words uttered before because nobody wants to be the harbinger of bad news.

The Practice Standards

Of course, other people have different takes on the CCSSM. In the Spring issue of the 2013 NCSM Newsletter, there is an interview with David Foster of the Silicon Valley Mathematics Initiative:

Q. What do you think is the best thing about the Common Core State Standards (CCSS)?

A. That's easy, the Standards for Mathematical Practice. These are the verbs of mathematics - what students should be doing while engaged with mathematics. I believe the practices will be the most influential aspect of the CCSS; their use will shift the mathematical thinking from the teacher to the students. If fully enacted the practices will have a dramatic impact on student learning.

It is *that* simple: The Practice Standards (**PS**) are there for the taking. No toil, no tears, and no sweat. Just "fully enact" them, and they will have *"a dramatic impact on student learning."*

Foster's answer represents the majority view. Nowadays, almost every meeting devoted to the CCSSM is consumed by incantations of the PS: *Read them, hang them up in every classroom, practice them when you teach*, and you will see the dramatic impact on student learning.

Eight Practice Standards (PS):

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

I will give you a different view: Discussions about the PS are not what teachers need unless such discussions are, by design, a prelude to content-based PD.

After teachers have replaced their knowledge of **TSM** with correct mathematics, then they can make sense of the PS.

When they only know **TSM**, a heavy intake of the PS may do more harm than good.

To explain this minority view, I will have to ask for your indulgence to listen to some mathematics.

I know this is against the grain: I should be talking about leadership, about securing funding, about team-building, about communicating with stakeholders, etc.

But I have to talk about mathematics, because math education is different from other kinds of education in that **mathematics** *is central to any discussion in math education.*

Most policy makers have not been made aware of this difference in their decision-making, but changes may be forthcoming.

I can illustrate the difference in the most superficial fashion possible in terms of the work of a substitute teacher.

Imagine that you are not particularly well-informed about either fractions or the Civil War. Which do you prefer: Sub for someone who wants you to teach about the Civil War the next day, or someone who wants you to teach the division of fractions?

I first learned about the dramatic difference between math education and reading education from the experts on reading when I was on the panel to write the NRC volume, *Preparing Teachers*, National Academy Press, 2010.

So please bear with me as I directly address some mathematical issues regarding the PS.

Two Examples.

Example 1. The Standard Algorithms.

These are the time-honored algorithms for computing the $+$, $-$, $\times$ and $\div$ of whole numbers over which the Math Wars were fought.

In the age of high-tech, what *mathematical* purpose do these algorithms serve?

In **TSM**, the four Standard Algorithms are four separate skills.

In the best of circumstances, there would be some explanation about each skill in terms of *place value*.

But children learn the algorithms mainly because

they are supposed to.

Much more is true, however. All four Standard Algorithms are built on one underlying principle: *if one knows how to compute with* **single digit numbers**, *then one can compute with any numbers no matter how big.*

This principle is part of a fundamental idea that pervades all of mathematics: **reduce the complex to the simple.**

Therefore learning the why and how of the Standard Algorithms is more than learning a skill. It is about learning a way of thinking that gets to the heart of mathematics.

Learning mathematics is a long journey. It is never too young to get children started on it.

For example, if students know how to add $2+4$, $3+5$, and $8+1$, then they can add $238+451$

$$\begin{array}{cccc} & 2 & 3 & 8 \\ + & 4 & 5 & 1 \\ \hline & 6 & 8 & 9 \end{array}$$

Reason for the algorithm:

$$\begin{aligned} 238+451 &= (200+30+8)+(400+50+1) \\ &= (200+400)+(30+50)+(8+1) \\ &= ((2+4)\times 100)+((3+5)\times 10)+(8+1) \end{aligned}$$

Moral: Instead of counting 451 steps from 238 (the **definition** of $238 + 451$)—which is error-prone—all they have to do is perform three simple single-digit additions: $2 + 4$, $3 + 5$, and $8 + 1$.

Make children aware of this, and they will be more motivated to learn the algorithm. They will also come to understand why they had to spend so much time doing single-digit additions in grades K-1.

The usual emphasis on "carrying" in teaching the addition algorithm is *emphasis misplaced.* The key idea is rather

> *the replacement of complex computations of addition by single-digit additions.*

Carrying becomes easier to learn when this basic idea has been firmly planted in children's minds.

Next, what is 43×26?

Definition. 43×26 is

$$\underbrace{26 + 26 + \cdots + 26}_{\text{43 times}}$$

Computing a product by the definition is very tedious in general. (Imagine computing 4321×26.)

What saves the day: Knowing how to multiply 4×2, 3×2, 4×6, and 3×6 is enough to do the multiplication 43×26.

Because of the lack of time, I will just write it down as a reminder:

$$
\begin{array}{rrrrr}
 & & & 4 & 3 \\
\times & & & 2 & 6 \\
\hline
 & & 2 & 5 & 8 \\
+ & & 8 & 6 & 0 \\
\hline
 & 1 & 1 & 1 & 8
\end{array}
$$

Instead of tedious additions, the multiplication 43×26 can be easily dispatched.

Now, suddenly, learning the multiplication table begins to make sense. It may even be fun!

TSM does not engage students in finding out the *purpose* of learning anything.

A similar discussion can be carried out about the subtraction and the long division algorithms.

Question: Why would the knowledge that the four algorithms are united by *"knowing single-digit computations implies knowing multi-digit computations"* improve student learning?

(1) It makes children see from the beginning that mathematics is not a laundry list of unrelated topics. Rather, it is an organic entity.

(2) Children appreciate simplicity in the face of complexity.

A clear mental framework improves learning.

(3) Children like to know the reason for doing something. They like a well-defined goal.

(Tell them that playing a video game is just an exercise that improves their hand-eye coordination, they'd lose interest. Tell them there is a game they can *win*, and they are hooked.)

Many students in middle school, or even high school, do not know the multiplication table.

Isn't it possible that many of them would have put greater effort into learning it if the importance of single-digit computations had been explained to them?

In **TSM**, memorizing the multiplication table is rammed down students' throats as a *purposeless* rote skill.

The relevant Practice Standards here:

2. Reason abstractly and quantitatively.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

After teachers have gone through the mathematical details of each algorithm, seeing how multi-digit computations boil down to single-digit computations, they can begin to abstract the reasoning (**PS 2 & 8**) and perceive the commonality among the four algorithms (**PS 7**).

Teachers will also learn that, without the the precise definitions (**PS 6**) of $+$, $-$, $\times$, and $\div$, it is impossible to appreciate virtue of each standard algorithm: it is a tremendous labor-saving device.

Then they can go back to their classrooms and teach their students with conviction and with purpose about the precision, reasoning and structure inherent in mathematics.

This is what the Practice Standards are about.

In other words, teachers appreciate the PS only *after* they have learned, *in a detailed and systematic way*, about the why and how of the Standard Algorithms from the new perspective.

Would a full day of general discussion about the PS, with teachers' knowledge of **TSM** intact, produce the desired understanding of the standard algorithms, and therefore *"a dramatic impact on student learning"?*

Highly unlikely.

Example 2. Equivalent fractions.

Are $\frac{2}{3}$ and $\frac{8}{12}$ equivalent fractions?

TSM tells us yes, they are, because:

$$\frac{2}{3} = \frac{2}{3} \times 1 = \frac{2}{3} \times \frac{4}{4} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Anything wrong with that?

Yes, it is **mathematically wrong** for the purpose this explanation is supposed to serve.

Because this answer may be even more puzzling than the original question, let me get down to the nitty gritty.

The concept of equivalent fractions has to be taken up more or less right at the beginning of any discussion of fractions. At that point, students know nothing about fractions except for the definition of a fraction.

How then to explain:

$$\frac{2}{3} = \frac{2}{3} \times 1 = \mathbf{\frac{2}{3}} \times \mathbf{\frac{4}{4}} = \mathbf{\frac{2 \times 4}{3 \times 4}} = \frac{8}{12} \; ?$$

An explanation can serve its purpose only if it is phrased in terms of things students already know. At this point, do they know how to multiply fractions? No. Therefore, what this explanation tells students is that $\frac{2}{3}$ is equal to $\frac{8}{12}$ because *there is some screwy stuff out there.*

This is not mathematics *education*.

The long-term effect of such an explanation on student *learning* can be devastating.

(1) If they are made to believe that $\dfrac{2}{3} \times \dfrac{4}{4} = \dfrac{2 \times 4}{3 \times 4}$ as a matter of principle (recall: they don't know what multiplication is), why not $\dfrac{2}{3} + \dfrac{4}{4} = \dfrac{2+4}{3+4}$?

(2) The concept of a fraction is the first *real* abstraction students face in their encounter with mathematics. They are already apprehensive. Such a sleight of hand in the midst of apprehension and uncertainty may turn them off to mathematics for good.

The basic PS involved here is Standard 7, *Look for and make use of structure*.

There a hierarchical structure in mathematics that is quite inflexible once we commit to a certain course of acton: some topics must precede others in the logical development, just as equivalent fractions must precede the multiplication of fractions (in the usual definition of multiplication).

Understanding the hierarchical structure of mathematics is much more subtle than the acquisition of a well-defined skill (e.g., how to carry out the long-division algorithm).

One does not understand how a democracy functions, for instance, by spending an afternoon reading through the Declaration of Independence and the American Constitution. *It takes immersion.*

One can appreciate the structure, and begins to see how the various topics are tightly interwoven in the hierarchy, only by *long and systematic exposure* to such a logical development.

TSM *usually ignores this hierarchical structure* (e.g., equivalent fractions precede fraction multiplication). Our teachers have been largely denied the opportunity to experience this structure.

When teachers, equipped only with **TSM**, get an afternoon of discussion on Practice Standard 7, will they realize that equivalent fractions cannot be explained in terms of fraction multiplication? Will they turn around and resolve to convey this sense of structure to their students?

Get real.

How then can we expect teachers to teach students about *mathematical structure* when their own exposure to this concept is by way of a half-day discussion on Practice Standard 7?

Reflections on the Two Examples

Successful implementation of the CCSSM requires a core group of teachers who

> have the content knowledge to meet the mathematical demands of the CCSSM, and
>
> have the pedagogical skills to convey this knowledge to students.

It would be fair to say that the content knowledge deficit is, *by far*, the more serious obstacle. Let us focus on that.

In the preceding two examples, we get to see the gulf between the abstract guiding principles known as the Practice Standards and the detailed content knowledge that gives meaning to PS.

There is no end of examples to further illustrate the gulf between the two:

- the correct symbolic expression for division-with-remainder,
- the meaning of *solving an equation*,
- the definition of fraction multiplication,
- the multiplication of decimals,
- the area formula of a rectangle,
- the conversion of a fraction to a decimal,
- the definition of congruence,
- the definition of similarity, etc., etc.

I hope I have given you some idea about **TSM**, PS, and the content knowledge teachers need in order to successfully implement the CCSSM.

More importantly, I hope you are convinced that the CCSSM are not the same old, same old, and that the CCSSM cannot be implemented by *tweaking teachers' content knowledge here and there* or exposing them to a few sessions on the PS.

There is no getting around it: teachers need

sustained, content-based PD.

Teachers have to *systematically* replace their knowledge of **TSM** with mathematics that is consonant with the PS.

The goal of the PD should be to provide teachers with this knowledge.

Before going further, let me take up an earlier remark: "If teachers only know **TSM**, then a heavy intake of the PS may do more harm than good."

PS 6 calls for precise definitions (in addition to precision in general). The mathematical purpose of having precise definitions is that they are the bedrock on which mathematical reasoning rests.

In mathematics, we make critical use of definitions for reasoning.

For example, we saw how, by comparing the *definition* of the multiplication of whole numbers and the multiplication algorithm, we came to realize the efficacy and the true value of the algorithm.

However, teachers who only know **TSM** consider a definition as "one more thing to memorize."

These teachers, if all they know about the Practice Standards is that they "will be the most influential aspect of the CCSSM", will make their students memorize definitions with greater zeal but *never use them in reasoning.*

Another example: PS #3 says that students should "Construct viable arguments and critique the reasoning of others."

A hallmark of **TSM** is that logical arguments are few and far between. Teachers who only know **TSM** are not used to "critiquing the reasoning of others", because such occasions rarely arise and because, most likely, they (the teachers) are not comfortable doing it and their colleagues are not comfortable receiving it.

Now if they are supposed to implement PS #3 in class, the most likely scenario is that they will allow their students to spend class time arguing with each other but will never render judgment as to who is right or who is wrong and, most importantly, why.

They also will not bring mathematical closure to open-ended discussions, because analysis of a discussion on the spot requires superior content knowledge.

After all, it is the “communication” among students that is important, isn’t it? Let students continue to “communicate”!

One can imagine scenarios of this type for every single one of the PS.

Professional Development

The following is worth repeating: *Through no fault of their own*, teachers only know **TSM**. They have not seen school mathematics done correctly, not in K–12, and not in institutions of higher learning.

This is why, right now, teachers generally equate mathematics with **TSM**.

Part of the difficulty with in-service PD is that there is not a good tradition there.

From a February 27, 2013 Education Week Article by Catherine Gewertz (*Teachers Say They Are Unprepared for Common Core*):

> "Due to resources, professional development is still the drive-by" variety in most districts, said the AFT's Ms. Dickinson.

In-service PD often means games, fun activities, new manipulatives, pedagogical strategies, and projects that you can directly bring back to your classroom.

Other times, it means making teachers feel good about themselves, and making them feel that they already know mathematics, or that mathematics can be learned without hard work.

Or, in recent years, *talking at length* about the PS as an end in itself.

Of course, there are better kinds of PD that discuss children's mathematical thinking, skillful use of technology, teacher-student communication, and refined teaching practices.

But the need for these good pedagogical practices pales in comparison with the overwhelming need for content knowledge.

Since we want sustained, content-based PD, the professional developers must possess the requisite content knowledge.

One would naturally turn to colleges and universities mathematicians for help with PD.

But not so fast, for three reasons.

(1) The college mathematics that mathematicians know is different from the school mathematics teacher need to know (in the same sense that algebra is different from geometry).

(2) Mathematicians are generally ignorant about school mathematics.

(3) We need *very* competent mathematicians to do PD.

Comments on (1).

There is a difference between college mathematics and school mathematics. For example, the CCSSM suggest that we define the $+$ and $\times$ between fractions in grade 5 by making use of the number line. Then the CCSSM suggest how to *derive from these definitions* the well-known formulas:

$$\frac{k}{\ell} + \frac{m}{n} = \frac{kn + m\ell}{\ell n} \quad \text{and} \quad \frac{k}{\ell} \times \frac{m}{n} = \frac{km}{\ell n}$$

However, college mathematics *defines* these operations on fractions by these same formulas:

$$\frac{k}{\ell} + \frac{m}{n} = \frac{kn + m\ell}{\ell n}, \quad \textit{by definition,}$$

$$\frac{k}{\ell} \times \frac{m}{n} = \frac{km}{\ell n}, \quad \textit{by definition.}$$

Simply put: School mathematics and college mathematics are different because *they have different starting points.*

Comments on (2).

There are reasons why mathematicians are generally ignorant about school mathematics.

One is that their work lies in the mathematical stratosphere and there is no inducement for them to spend time on ground-level school mathematics. (*You can't get people who proved the existence of the Higgs boson to think about how to improve the iPhone.*)

Another is the long separation between schools of education and departments of mathematics on university campuses.

Yet another is the perception of most research mathematicians that school education is a bottomless pit in which intellectual merits matter very little.

The end result is that ignorance about school mathematics prevails in the math community.

Comments on (3).

Why do we need “very competent” mathematicians to do PD?

Because the CCSSM are trying to lead the nation out of the **TSM** jungle.

Given that **TSM** has ruled school mathematics education for decades, getting rid of **TSM** requires a steady guiding hand. Such guidance can only come from a deep knowledge of mathematics.

A common misconception is that **any** mathematics professor is a content expert.

I was given tenure at UC Berkeley in 1968, but looking back, I now *know* it would have been a mistake for anyone to consult me about K-12 math education back then. *I didn't have the breadth of knowledge to make sound decisions.*

There is no substitute for good judgment in choosing mathematics consultants.

It should be obvious that I want mathematicians to play a significant role in the implementation of the CCSSM. But in the last five minutes or so I seemed to be doing my best to talk you out of it.

This is easy to explain: If you want to build a skyscraper, you have to hire an architect. Not *any* architect, but the best architect available. So you understand why I point out all the possible pitfalls in choosing this architect.

Allow me to repeat: We must do all we can to satisfy teachers' overwhelming need for content knowledge that is

mathematically correct and

suitable for use in the appropriate grades.

A Proposal

We accept the fact that, because of the heavy content component, the kind of PD we need requires professional mathematicians to take the lead.

Is there a concrete strategy in PD that can produce a corps of mathematically knowledgeable teachers?

Realistically, it may be too difficult to coordinate a *national* effort for this kind of PD, and a school district may not have enough resources to get it done.

Let us speculate on how a *state* might try to meet this challenge.

The overall strategy must be conservative in estimating *how many* teachers we can reach directly, and *how long* it will take.

It must recognize the reality that there are not enough knowledgeable mathematicians who are familiar with school mathematics. *Recall:* school mathematics is not college mathematics.

Providing PD for **all** *elementary* teachers is not an option. The use of *math specialists in grades 4–6* (teachers who only teach math) will be a necessity and we should limit the PD to math specialists in elementary schools.

For math specialists and for all math teachers in middle school and high school: A manageable statewide project is to focus on producing a nucleus of teachers with a robust content knowledge.

First stage: Provide *intensive content-based* PD to a very select group, geographically well distributed, and repeat for several years. Make these the **nucleus**.

Let each "nucleus teacher" be a math coach for his/her own local district(s).

Second stage: Use “nucleus teachers” to provide PD to local teachers. Each such PD session should be the collaborative effort of two or more “nucleus teachers”. Concurrently, the state sends a group of “roving ambassadors” to give feedback and assistance to the “nucleus teachers” around the state.

The “roving ambassadors” provide resources and maintain quality control.

Each “roving ambassadors” unit would consist of both mathematicians and educators/master teachers.

Such a collaboration is essential, at least initially. It will ensure that the content of the PD is

> *mathematically correct and*
>
> *suitable for use in the appropriate grades.*

Finding the right people, especially the right mathematicians, to do such important work is a serious issue. This is where good administrative judgment comes in. This is where leadership matters.

I cannot make things easy for you.

My belief is that we have wasted precious time in implementing the CCSSM. But I also beieve it is not too late.

The CCSSM represent the first opportunity in decades to make school mathematics learnable, but implementing the CCSSM requires that we have *mathematically* knowledgeable teachers.

To this end, we have to commit to a multiyear project.

Rather than thinking of reasons why this can't be done, think instead: *Do we have a choice?*

The high-profile volume, *Rising Above the Gathering Storm* (2007), envisions the end of American leadership in science and technology before 2050. It makes four recommendations for change, and the first is:

Increase America's talent pool
by vastly improving K-12 science
and mathematics education.

The recommended action of highest priority is to **"place knowledgeable math and science teachers in the classroom."**

Do we have a choice?

A journey of ten thousand miles begins with a single step. We have waited far too long to begin the journey.

Let us take the first step *now*.